The Order of Operations

Algebra can be thought of as a game. When you know the rules, you have a much better chance of winning! In addition to knowing how to add, subtract, multiply, and divide integers, fractions, and decimals, you must also use the **order of operations** correctly.

Although you have previously studied the rules for *order of operations*, here is a quick review.

Rules for Order of Operations

Always start on the *left* and move *to* the *right*.

1.	Do operations inside *grouping symbols* first.	( ) , [ ], **or** $\frac{x}{y}$
2.	Then do all *powers* (exponents) **or** *roots*.	x^2 **or** $\sqrt{x}$
3.	Next do *multiplication* **or** *division*—as they occur from left to right.	• **or** ÷
4.	Finally, do *addition* **or** *subtraction*—as they occur from left to right.	+ **or** –

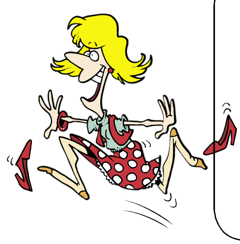
Remember: The fraction bar is considered a **grouping symbol**.

Example: $\frac{3x^2 + 8}{2} = (3x^2 + 8) \div 2$

Note: In an **expression** where more than one set of *grouping symbols* occurs, work within the innermost set of symbols first, then work your way outward.

The order of operations makes sure everyone doing the problem correctly will get the same answer.

Some people remember these rules by using this mnemonic device to help their memory.



Please **P**ardon **M**y **D**ear **A**unt **S**ally*

Please .... **P**arentheses (grouping symbols)
Pardon.................... **P**owers
My **D**ear.................. **M**ultiplication or **D**ivision
Aunt **S**ally............... **A**ddition or **S**ubtraction

*Also known as **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally—**P**arentheses, **E**xponents, **M**ultiplication or **D**ivision, **A**ddition or **S**ubtraction.

Remember: You do multiplication **or** division—*as they occur* from *left to right*, and then addition **or** subtraction—*as they occur* from *left to right*.

Study the following.

$$25 - 3 \cdot 2 =$$

There are no grouping symbols. There are no **powers (exponents)** or **roots**. We look for multiplication or division and find multiplication. We multiply. We look for addition or subtraction and find subtraction. We subtract.

$$25 - 3 \cdot 2 =$$
$$25 - 6 =$$
$$19$$

Study the following.

$$12 \div 3 + 6 \div 2 =$$

There are no grouping symbols. There are no *powers* or *roots*. We look for multiplication or division and find division. We divide. We look for addition or subtraction and find addition. We add.

$$12 \div 3 + 6 \div 2 =$$
$$4 + 3 =$$
$$7$$

If the rules were ignored, one might divide 12 by 3 and get 4, then add 4 and 6 to get 10, then divide 10 by 2 to get 5—which is the wrong answer. Agreement is needed—using the agreed-upon *order of operations*.

Study the following.

$30 - 3^3 =$

There are no grouping symbols. We look for powers and roots and find powers, 3^3. We calculate this. We look for multiplication or division and find none. We look for addition or subtraction and find subtraction. We subtract.

$30 - 3^3 =$
$30 - 27 =$
3

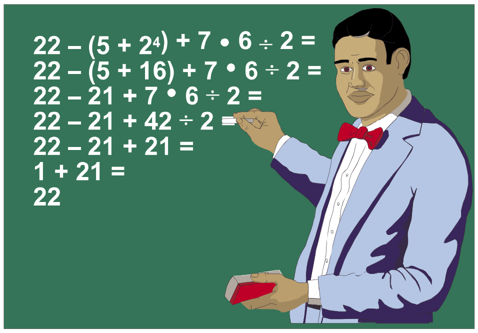
Study the following.

$22 - (5 + 2^4) + 7 \bullet 6 \div 2 =$

We look for grouping symbols and see them. We must do what is inside the parentheses first. We find addition and a power. We do the power first and then the addition. There are no roots. We look for multiplication or division and find both. We do them in the order they occur, left to right, so the multiplication occurs first. We look for addition or subtraction and find both. We do them in the order they occur, left to right, so the subtraction occurs first.

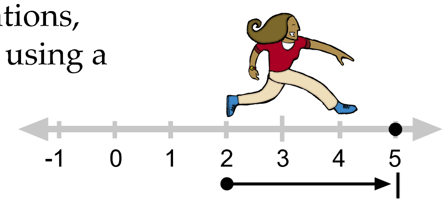
Please	**P**arentheses
Pardon	**P**owers
My	**M**ultiplication *or*
Dear	**D**ivision
Aunt	**A**ddition *or*
Sally	**S**ubtraction

$22 - (5 + 2^4) + 7 \bullet 6 \div 2 =$
$22 - (5 + 16) + 7 \bullet 6 \div 2 =$
$22 - 21 + 7 \bullet 6 \div 2 =$
$22 - 21 + 42 \div 2 =$
$22 - 21 + 21 =$
$1 + 21 =$
22



Adding Numbers by Using a Number Line

After reviewing the rules for order of operations, let's get a visual feel for adding integers by using a **number line**.

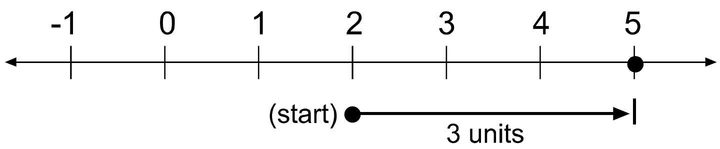


Example 1

Add 2 + 3

1. Start at 2.
2. Move 3 units to the right in the *positive* direction.
3. Finish at 5.

   So, 2 + 3 = 5.

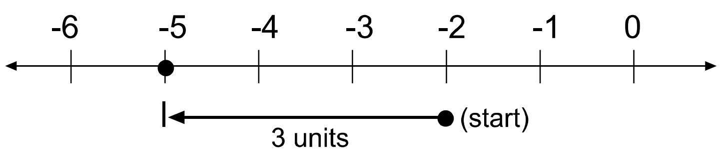


Example 2

Add -2 + (-3)

1. Start at -2.
2. Move 3 units to the left in the *negative* direction.
3. Finish at -5.

   So, -2 + (-3) = -5.

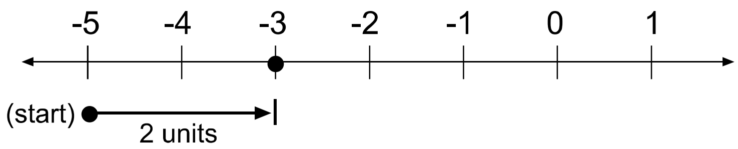


Example 3

Add -5 + 2

1. Start at -5.
2. Move 2 units to the right in a *positive* direction.
3. Finish at -3.

So, -5 + 2 = -3.

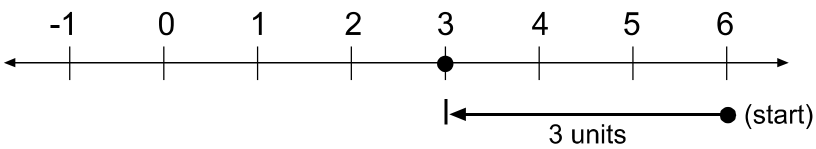


Example 4

Add 6 + (-3)

1. Start at 6.
2. Move 3 units to the left in a *negative* direction.
3. Finish at 3.

So, 6 + (-3) = 3.



Addition Table

Look for *patterns* in the Addition Table below.

Addition Table

+	4	3	2	1	0	-1	-2	-3	-4
4	8	7	6	5	4	3	2	1	0
3	7	6	5	4	3	2	1	0	-1
2	6	5	4	3	2	1	0	-1	-2
1	5	4	3	2	1	0	-1	-2	-3
0	4	3	2	1	0	-1	-2	-3	-4
-1	3	2	1	0	-1	-2	-3	-4	-5
-2	2	1	0	-1	-2	-3	-4	-5	-6
-3	1	0	-1	-2	-3	-4	-5	-6	-7
-4	0	-1	-2	-3	-4	-5	-6	-7	-8

addends (any numbers being added)

sums (the result of adding numbers together)

addends

sums

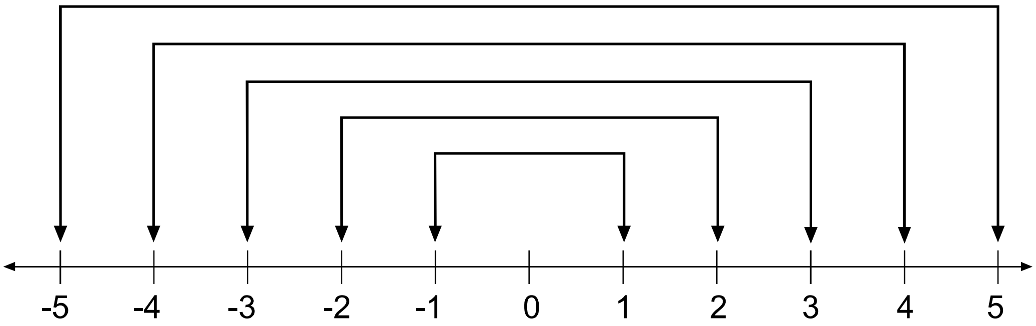
- Look at the *positive sums* in the table. Note the *addends* that result in a positive sum.
- Look at the *negative sums* in the table. Note the *addends* that result in a negative sum.
- Look at the sums that are *zero*. Note the *addends* that result in a sum of zero.
- **Additive Identity Property**—when zero is added to any number, the sum is the number. Note that this property is true for addition of integers.
- **Commutative Property of Addition**—the order in which numbers are added does *not* change the sum. Note that this property is true for addition of integers.
- **Associative Property of Addition**—the way numbers are grouped when added does *not* change the sum. Note that this property is true for addition of integers.

Opposites and Absolute Value

Although we can visualize the process of adding by using a number line, there are faster ways to add. To accomplish this, we must know two things: *opposites* or *additive inverses* and **absolute value**.

Opposites or Additives Inverses

5 and -5 are called *opposites*. Opposites are two numbers whose points on the number line are the same distance from 0 but in opposite directions.



Every **positive integer** can be paired with a **negative integer**. These pairs are called *opposites*. For example, the opposite of 4 is -4 and the opposite of -5 is 5.

The opposite of 4 can be written -(4), so -(4) equals -4.

$$-(4) = -4$$

The opposite of -5 can be written -(-5), so -(-5) equals 5.

$$-(-5) = 5$$

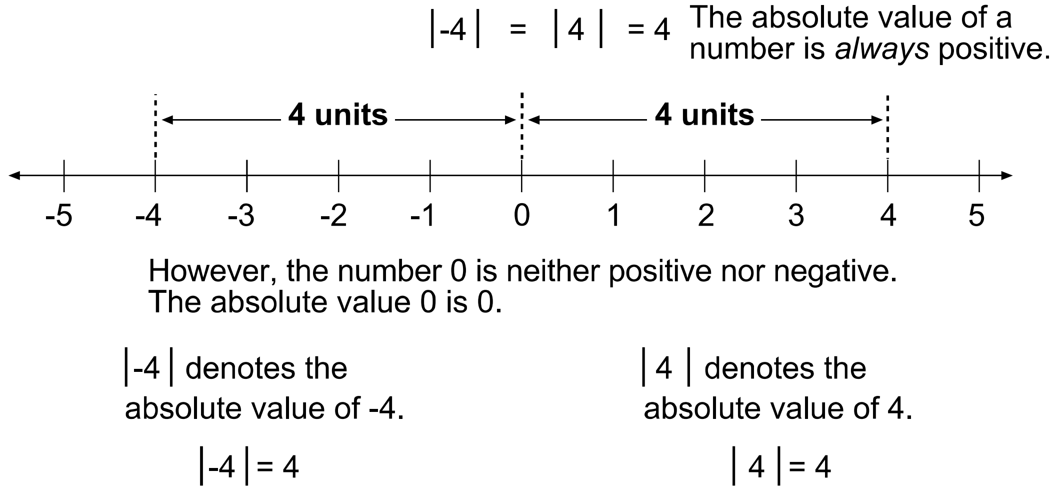
Two numbers are opposites or *additive inverses* of each other if their sum is zero.

For example: $4 + -4 = 0$
$-5 + 5 = 0$

Absolute Value

The *absolute value* of a number is the distance the number is from the *origin* or zero (0) on a number line. The symbol | | placed on either side of a number is used to show absolute value.

Look at the number line below. -4 and 4 are different numbers. However, they are the same distance in number of units from 0. Both have the same *absolute value* of 4. Absolute value is *always* positive because distance is always positive—you cannot go a negative distance. The absolute value of a number tells the number's *distance* from 0, not its *direction*.



The absolute value of 10 is 10. We can use the following notation.

$|10| = 10$

The absolute value of -10 is also 10. We can use the following notation.

$|-10| = 10$

Both 10 and -10 are 10 units away from the origin. So, the absolute value of both numbers is 10.

The absolute value of 0 is 0.

$|0| = 0$

The *opposite* of the absolute value of a number is *negative*.

$-|8| = -8$

Now that we have this terminology under our belt, we can introduce two rules for adding numbers which will enable us to add quickly.

Adding Positive and Negative Integers

There are specific rules for adding positive and negative numbers.

1. **If the two integers have the *same sign*, *add* their absolute values, and *keep* the *sign*.**

   Example
   -5 + (-7)

Think: Both integers have the same signs and the signs are negative. Add their absolute values.

| -5 | = 5

| -7 | = 7

5 + 7 = 12

Keep the sign. The sign will be negative because both signs were negative. Therefore, the answer is -12.

-5 + -7 = -12

2. **If the two integers have *opposite signs*, *subtract* the absolute values. The answer has the *sign* of the integer with the *greater* absolute value.**

Example

-8 + 3

Think: Signs are opposite. Subtract the absolute values.

$|-8| = 8$

$|3| = 3$

8 – 3 = 5

The sign will be negative because -8 has the greater absolute value. Therefore, the answer is -5.

-8 + 3 = -5

Example

-6 + 8

Think: Signs are opposite. Subtract the absolute values.

$|-6| = 6$

$|8| = 8$

8 – 6 = 2

The sign will be positive because 8 has a greater absolute value. Therefore, the answer is 2.

-6 + 8 = 2

Example
5 + (-7)

Think: Signs are opposite. Subtract the absolute values.

| 5 | = 5

| -7 | = 7

7 – 5 = 2

The sign will be negative because -7 has the greater absolute value. Therefore, the answer is -2.

5 + -7 = -2

Rules for Adding Integers

• The sum of two positive integers is *positive*.	(+) + (+) = +
• The sum of two negative integers is *negative*.	(-) + (-) = -
• The sum of a positive integer and a negative integer takes the *sign of the number with the greater absolute value*.	(-) + (+) = (+) + (-) = } use sign of number with greater absolute value
• The sum of a positive integer and a negative integer is zero if numbers have the *same absolute value*.	(*a*) + (-*a*) = 0 (-*a*) + (*a*) = 0

Check Yourself Using a Calculator When Adding Positive and Negative Integers

Use a **calculator** with a [+/-] **sign-change** key.

For example, for -16 + 4, you would enter (for most calculators) 16 [+/-] [+] 4 [=] and get the answer -12.

Subtracting Integers

In the last section, we saw that 8 plus -3 equals 5.

$$8 + (-3) = 5$$

We know that 8 minus 3 equals 5.

$$8 - 3 = 5$$

Below are similar examples.

$10 + (-7) = 3$	$12 + (-4) = 8$
$10 - 7 = 3$	$12 - 4 = 8$

These three examples show that there is a connection between adding and subtracting. As a matter of fact, we can make any subtraction problem into an addition problem and any addition problem into a subtraction problem.

This idea leads us to the following definition.

Definition of Subtraction
$\boldsymbol{a - b = a + (-b)}$

Examples

$$8 - 10 = 8 + (-10) = -2$$

$$12 - 20 = 12 + (-20) = -8$$

$$-2 - 3 = -2 + (-3) = -5$$

Even if we have $8 - (-8)$, this becomes

8 plus the opposite of -8, which equals 8.

$$8 + [-(-8)] =$$

$$8 + 8 = 16$$

And -9 – (-3), this becomes

-9 plus the opposite of -3, which equals 3.

-9 + -(-3)

-9 + 3 = -6

Shortcut Two negatives become one positive!

10 – (-3) becomes 10 plus 3.

10 + 3 = 13

And -10 – (-3) becomes -10 plus 3.

-10 + 3 = -7

Generalization for Subtracting Integers

Subtracting an integer is the same as adding its opposite.

$a - b = a + (-b)$

✓ Check Yourself Using a Calculator When Subtracting Negative Integers

Use a **calculator** with a [+/-] **sign-change** key.

For example, for 18 – (-32), you would enter
18 [–] 32 [+/-] [=] and get the answer 50.